Institut für Informatik
Norbert Blum, Marek Karpinski,
Rolf Klein, Heiko Röglin

Pearls of Algorithms
Winter 2012/13

Problem Set 2.3

Problem 1

We modify the FastCut algorithm by setting $t = \lceil 1 + 3n/4 \rceil$ instead of $t = \lceil 1 + n/\sqrt{2} \rceil$. Then fewer edges are contracted in each recursive call of FastCut and one can show that the success probability of a single iteration of the FastCut algorithm increases to a constant $c > 0$. What is the running time of this algorithm?

Problem 2

We roll a standard fair die over and over. What is the expected number of rolls until the first pair of consecutive sixes appears?

Problem 3

Let X and Y be independent, uniform random variables on $[0, 1]$. Find the density function and distribution function for $X + Y$.

Problem 4

Let $X_1, \ldots, X_n$ be independent random variables with density functions $f_1, \ldots, f_n$. Furthermore, let $f_i(x) \leq \phi$ for every $i \in \{1, \ldots, n\}$ and every $x \in \mathbb{R}$. Give an upper bound for the probability of $X_1 + \ldots + X_n \in [a, a + \varepsilon]$, where $a \in \mathbb{R}$ and $\varepsilon > 0$ are fixed arbitrarily.

Problem 5

Let n points be placed uniformly at random on the boundary of a circle of circumference 1. These n points divide the circle into n arcs. Let x denote an arbitrary fixed point on the circle?

(a) What is the average arc length?

(b) What is the expected length of the arc that contains the point x?