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Pearls of Algorithms
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Problem Set 2.2

Problem 1

Let X and Y be two sets that are chosen independently and uniformly at random from all the 2^n subsets of $\{1, \ldots, n\}$. Determine $\mathbf{Pr}(X \subseteq Y)$ and $\mathbf{Pr}(X \cup Y = \{1, \ldots, n\})$.

Problem 2

A graph can have several min-cuts. Using the analysis of the contract algorithm, prove that there can be at most $n(n-1)/2$ distinct min-cuts.

Problem 3

An r-way cut of a graph $G = (V, E)$ is a set $C \subseteq E$ of edges such that the graph $(V, E \setminus C)$ has at least r distinct connected components. Adapt the contract algorithm to find a minimum r-way cut and bound the probability that it succeeds in one iteration.

Problem 4

A monkey types on a 26-letter keyboard that has lowercase letters only. Each letter is chosen independently and uniformly at random from the alphabet. If the monkey types 1,000,000,000 letters, what is the expected number of times the sequence "banana" appears?

Problem 5

Consider the following recursive algorithm.

```
RandomRecursion(ℓ)
{
    Print ℓ.
    Toss a fair coin.
    If the coin is heads call RandomRecursion(ℓ + 1).
    Toss a fair coin.
    If the coin is heads call RandomRecursion(ℓ + 1).
}
```

(a) What is the probability that `RandomRecursion`(0) terminates?

(b) How often does `RandomRecursion`(0) output a particular number $k \in \mathbb{N}$ in expectation?

(c) How many numbers does `RandomRecursion`(0) output in expectation?