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Pearls of Algorithms
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Problem Set 2.3

Problem 1

A monkey types on a 26-letter keyboard that has lowercase letters only. Each letter is chosen independently and uniformly at random from the alphabet. If the monkey types 1,000,000,000 letters, what is the expected number of times the sequence "banana" appears?

Problem 2

Consider the following recursive algorithm.

```
RandomRecursion(ℓ)
{
    Print ℓ.
    Toss a fair coin.
    If the coin is heads call RandomRecursion(ℓ + 1).
    Toss a fair coin.
    If the coin is heads call RandomRecursion(ℓ + 1).
}
```

(a) What is the probability that `RandomRecursion`(0) terminates?

(b) How often does `RandomRecursion`(0) output a particular number $k \in \mathbb{N}$ in expectation?

(c) How many numbers does `RandomRecursion`(0) output in expectation?

Problem 3

Consider the following variation of the coupon collector's problem. Each box of cereal contains one of kn different coupons. The coupons are organized into n groups of k coupons each. The coupons $1, .., k$ constitute the first group, the coupons $k+1, \ldots, 2k$ constitute the second group, and so on. Once you obtain one coupon from every group, you can obtain a prize. Assuming that the coupon in each box is chosen independently and uniformly at random from the kn possibilities, what is the expected number of boxes you must buy before you can claim the prize?

Problem 4

We role a standard fair die over and over. What is the expected number of rolls until the first pair of consecutive sixes appears?