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Pearls of Algorithms
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Problem Set 2.2

Problem 1

Let X and Y be two sets that are chosen independently and uniformly at random from all the 2^n subsets of $\{1, \ldots, n\}$. Determine $\mathbf{Pr}(X \subseteq Y)$ and $\mathbf{Pr}(X \cup Y = \{1, \ldots, n\})$.

Problem 2

A graph can have several min-cuts. Using the analysis of the contract algorithm, prove that there can be at most $n(n-1)/2$ distinct min-cuts.

Problem 3

An r-way cut of a graph $G = (V, E)$ is a set $C \subseteq E$ of edges such that the graph $(V, E \setminus C)$ has at least r distinct connected components. Adapt the contract algorithm to find a minimum r-way cut and bound the probability that it succeeds in one iteration.

Problem 4

We modify the FastCut algorithm by setting $t = \lceil 1 + 3n/4 \rceil$ instead of $t = \lceil 1 + n/\sqrt{2} \rceil$. Then fewer edges are contracted in each recursive call of FastCut and one can show that the success probability of a single iteration of the FastCut algorithm increases to a constant $c > 0$. What is the running time of this algorithm?