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Pearls of Algorithms
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Problem Set 2.1

Problem 1

We roll a standard six-sided die independently n times. What is the probability of the following events?

(a) All dice show the same number

(b) The sum of the dice is a multiple of 3.

(c) The product of the dice is even.

Problem 2

We flip a fair coin n times. For $k \in \mathbb{N}$, find an upper bound on the probability that there is a sequence of $\lceil \log_2 n \rceil + k$ consecutive heads. Your bound should decrease exponentially in k.

Problem 3

Give an example of three events that are pairwise independent but not mutually independent.

Problem 4

In a quiz show three participants can win a trip to Hawaii if they win the following game: Each participant gets independently and uniformly at random either a red or a green hat; he cannot see the color of his hat but the colors of the others. Then, without communicating, all three write down either "red", "green" or, "unknown". The three players win if at least one player wrote down "red" or "green" and if all players that wrote down "red" or "green" correctly guessed the color of their own hat.

(a) Give a strategy for the three players that guarantees a chance for winning of exactly 50%.

(b) Is there a scheme that guarantees a winning probability of more than 50%?

Problem 5

In a quiz show 100 participants $P_1, \ldots, P_{100}$ can win a trip to Hawaii. The host of the show sets up a room with 100 closed boxes. Every box contains a sheet with a unique number from $\{1, 2, \ldots, 100\}$, and the boxes are placed one after another in a random order in the room. The participants, who can agree upfront on a strategy but cannot communicate anymore once the show has started, enter the room one after another.

The first participant P_1 enters the room first. He can open 50 of the boxes. If none of these boxes contains his own number 1, then all participants lose. Otherwise, he closes all the boxes again and leaves the room. He is not allowed to reorder the boxes or to leave any other hints.

Then the second participant P_2 enters the room. He can also open 50 of the boxes. If none of these boxes contains his own number 2, then all participants lose. Otherwise, he closes all the boxes again and leaves the room. He is not allowed to reorder the boxes or to leave any other hints. This is continued until the last participant P_{100}.

Hence, the participants win if and only if each of them opens the box with his own number. If there is a single participant who does not open the box with his own number, everybody loses. Advise the participants on a strategy and compute the probability of winning according to your strategy.

website of the lecture: `http://www.roeglin.org/teaching/WS2011/Pearls.html`