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Pearls of Algorithms
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Problem Set 3.1

Problem 1 (Linear Programming)

A gardener has a plot of land of 100 m^2 that she wishes to use to plant flowers and vegetables.

- She has a total budget of €720. Labor and material costs for planting are €6 per m^2 for vegetables and €9 per m^2 for flowers.
- The revenue she gets from selling the produce is €10 per m^2 for vegetables and €20 per m^2 for flowers.
- Her experience tells her that only 60 m^2 of her plot of land are suitable for growing flowers; the other 40 m^2 can only be used for growing vegetables.
- She has 5kg of fertilizer, and she needs 40g per m^2 for growing flowers and 60g per m^2 for growing vegetables.

(a) Formulate a linear program for optimizing the gardener's profit.

(b) Solve the linear program graphically. That is, draw the polytope of feasible solutions and identify the optimal vertex.

Problem 2 (Expected Values)

(a) Suppose n drunken sailors return to their ship, and each of them chooses one of the n cabins uniformly at random. What is the expected number of sailors that sleep in their own cabin?

(b) How often do you need to toss a normal dice in expectation until you get a six?

Problem 3 (Probabilities)

In a quiz show three participants can win a trip to Hawaii if they win the following game: Each participant gets independently and uniformly at random either a red or a green hat; he cannot see the color of his hat but the colors of the others. Then, without communicating, all three write down either "red", "green" or, "unknown". The three players win if at least one player wrote down "red" or "green" and if all players that wrote down "red" or "green" correctly guessed the color of their own hat.

(a) Give a strategy for the three players that guarantees a chance for winning of exactly 50%.

(b) Is there a scheme that guarantees a winning probability of more than 50%?

Problem 4 (Nemhauser/Ullmann Algorithm)

Provide an example with n items on which the Nemhauser/Ullmann algorithm for the knapsack problem needs time $\Omega(2^n)$.

website of the lecture: `http://www.roeglin.org/teaching/WS2010/Pearls.html`