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Problem Set 7

Let $G = (V, E)$ be a simple directed graph, $u \colon E \to \mathbb{R}_{>0}$ be a capacity function on the edges, $b \colon V \to \mathbb{R}$ be a balance function on the nodes, and the costs $c_e \in [0, 1]$ of an edge $e \in E$ be drawn at random according to a density function bounded from above by $\phi \geq 1$. As in the lecture, we introduce two auxiliary nodes s and t and up to n auxiliary edges such that s is the only source and t is the only sink in the flow network. We want to bound the number of iterations of the SSP algorithm on this instance.

Problem 1

Let $E^{-1} := \{(w, v) : (v, w) \in E\}$. For given $\varepsilon > 0$, find an upper bound for the probability that there are two distinct nodes v, w and two distinct v-w paths P_1, P_2 in $(V, E \,\dot\cup\, E^{-1})$ with $|c(P_1) - c(P_2)| \leq \varepsilon$ which depends linearly on ε.

Problem 2

Suppose that every two distinct possible s-t paths in any residual network have different lengths. Show that the number of iterations the SSP algorithm needs is bounded by 3^{m+n}.

Problem 3

Suppose that for all $\varepsilon > 0$ and for all $t \in \mathbb{R}$, the probability that the SSP algorithm uses an s-t path whose length is in $[t, t + \varepsilon]$ is bounded by $a \cdot \varepsilon \cdot \phi$ for some $a > 0$. Use Problem 1 and Problem 2 to show that the expected number of iterations is bounded from above by $a \cdot n \cdot \phi$.

Hint: The assumption of Problem 2 holds with probability 1. The bound 3^{m+n} can therefore be interpreted as the „worst case.“