Institut für Informatik
Prof. Dr. Heiko Röglin
Magdalena Reichel

Probabilistic Analysis
of Algorithms
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Problem Set 5

Problem 1

Let a graph $G = (V, E)$ and a function $\ell : E \to \mathbb{R}_{>0}$ that assigns a length to each edge be given. Let $s \neq t$ be two vertices of G. For any path $\pi = (s = v_0, v_1, ..., v_{k-1}, t = v_k)$ let $\ell(\pi) := \sum_{i=0}^{k-1} \ell(v_i, v_{i+1})$, i.e. the length of a path is the sum of the lengths of the edges it contains.

a) Find an algorithm that computes the shortest path from s to t via dynamic programming. What is the worst-case running time of this algorithm?

b) Enhance the setting from above by an additional function $c : E \to \mathbb{R}_{>0}$ that assigns a cost to each edge. Again the cost of a path equals the sum of the costs of its edges. We want to minimize the length of the path as well as its total cost. In general there is no path optimizing both criteria simultaneously and we are interested in the set of Pareto-optimal paths.
Give an algorithm to find the set of Pareto-optimal paths and analyze its worst-case and smoothed running time.

Problem 2

Let I be an instance of the knapsack problem as usual, but now you are allowed to pack each item up to k times. How can this problem be reduced to an instance of the classical knapsack problem with $n \cdot \mathcal{O}(\log k)$ many items?

Problem 3

Consider a modification of the knapsack problem where there are multiple profits for each object. This means that instead of one vector $p \in \mathbb{R}^n$ describing the profits there are multiple vectors $p^1 \dots p^n$ and you want to maximize all $p^i \cdot x$. Is it possible to generalize the Nemhauser/Ullmann algorithm to find the set of Pareto-optimal solutions for this modified problem?

Problem 4

Let X be the set of feasible solutions for an optimization problem defined by two functions $a : X \to \mathbb{R}$ and $b : X \to \mathbb{R}$, where the task is to maximize $a(x)$ and $b(x)$. Assume you have a fast implementation of an algorithm that computes the set of Pareto-optimal solutions $\mathcal{P}(X)$. How does it help you to find the convex hull of the set $S = \{(a(x), b(x)) \mid x \in X\}$?