Institut für Informatik
Prof. Dr. Heiko Röglin
Magdalena Reichel

Probabilistic Analysis
of Algorithms
Summer 2013

Problem Set 2

Problem 1

Let X and Y be independent, uniform random variables on $[0, 1]$. Find the density function and distribution function for $X + Y$.

Problem 2

Let $X_1, \ldots, X_n$ be independent random variables with density functions $f_1, \ldots, f_n$ and let $\lambda_1, \ldots \lambda_n \in \mathbb{R}_+$ be arbitrary. Furthermore, let $f_i(x) \leq \phi$ for every $i \in \{1, \ldots, n\}$ and every $x \in \mathbb{R}$. Give an upper bound for the probability of $\lambda_1 X_1 + \ldots + \lambda_n X_n \in [a, a + \varepsilon]$, where $a \in \mathbb{R}$ and $\varepsilon > 0$ are fixed arbitrarily.

Problem 3

We agree to try to meet between 12 and 1 for lunch at our favorite sandwich shop. Because of our busy schedules, neither of us is sure when we'll arrive; we assume that, for each of us, our arrival time is uniformly distributed over the hour. So that neither of us has to wait too long, we agree that we will each wait exactly 15 minutes for the other to arrive, and then leave. What is the probability we actually meet each other for lunch?

Problem 4

Let n points be placed uniformly at random on the boundary of a circle of circumference 1. These n points divide the circle into n arcs.

(a) What is the average arc length?

(b) Let x denote an arbitrary fixed point on the circle. What is the expected length of the arc that contains the point x?